Sound Transmission in an Extended Tube Resonator

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Abstract

The transmission of sound in a duct with sudden area expansion and extended inlet is investigated in the case the walls of the duct lying in overlap region lined with different acoustically absorbent materials. By using the series expansion in overlap region and using Fourier transform technique elsewhere we obtain a Wiener-Hopf equation whose solution involve a set of infinitely many unknown expansion coefficients satisfying a system of linear algebraic equations. Numerical solution of this system is obtained by truncating the infinite series and then the variation of transmission coefficient for different values of problem parameters are displayed graphically.

Key words: Sound transmission, absorbent lining, extended inlet

1. Introduction

It is possible to reduce the unwanted noise propagating along a duct by using a reactive or a dissipative silencer. In reactive silencers sudden area changes in cross sectional area help to reduce the energy in the transmitted wave via internal reflections. Simple expansion chambers work in accordance with this principle and widely investigated in literature [1], [2], [3], [4]. In further investigations it has been shown that the extension of inlet and outlet tubes into the expansion chamber increased the effciency in noise reduction [5],[6],[7].

It has also been proved that the treatment of the duct walls with acoustically absorbent lining is another effective method in reducing unwanted noise [8]. Combining the two effective approaches in noise reduction, transmission properties of a combination silencer consist of an expansion chamber whose walls are treated by acoustic liners has been analysed in [9].

In this paper, the transmission of sound in an extended tube resonator whose walls in overlap region, where extended inlet and expanding duct walls overlap, are treated by different locally reacting lining is investigated. So the main objective of this paper is to reveal the influence of the partial lining on the transmitted field and to present an alternative method of formulation. The method adopted in this paper consists of expanding the field in the overlap region into a series of complete set of orthogonal eigenfunctions and using the Fourier transform technique elsewhere. The problem is then reduced directly into a Wiener-Hopf equation whose solution involve a set of infinitely many unknown expansion coefficients satisfying a system of linear algebraic equations. Numerical solution to these systems are obtained for various values of the parameters of the problem such as the radii of the semi infinite waveguides, the overlap length and the impedance loading whereby the effects of these parameters on the transmitted field are presented graphically.

The time dependence is assumed to be exp(-iωt) with ω being the angular frequency and suppressed throughout this paper.

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2. Materials and Method

Consider two opposite semi-infinite circular cylindrical waveguides of different radii with common longitudinal axis, say z, in a cylindrical polar coordinate system (ρ,ϕ,z). They occupy the regions $\rho = a$; $z < l$ and $\rho = b > a$; $z > 0$; respectively, where l represents the overlap length. These two waveguides are connected with a vertical wall at $a < \rho < b$, $z = 0$. The parts of the surfaces $\rho = a+0$ and $\rho = b - 0$ lying in the overlap region $0 < z < l$ of the waveguides are assumed to be treated by acoustically absorbing linings which are characterized by constant but different surface impedances respectively, while the remaining parts are perfectly rigid (see Fig. 1). The waveguides are immersed in the inviscid and compressible stationary fluid of density ρ_0 and sound speed c. A plane sound wave is incident from the positive z-direction, through the waveguide of radius $\rho = a$. From the symmetry of the geometry of the problem and the incident field the scattering field everywhere will be independent of the ϕ coordinate. We shall therefore introduce a scalar potential $u(\rho,z)$ which defines the acoustic pressure and velocity by $p = i\omega\rho_0 u$ and $\mathbf{v}$ = grad u, respectively.

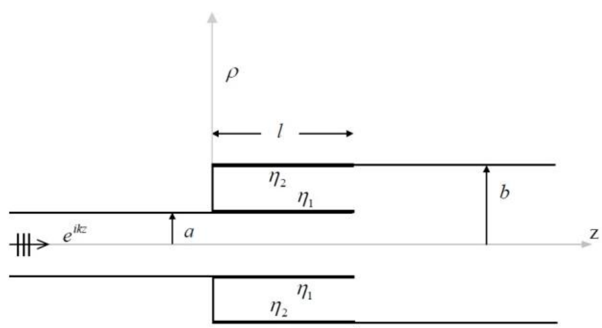

Figure 1. Geometry of the problem

It is convenient to write the total field in different regions as:

$$u^T(r,z) = \begin{cases} u_1(\rho,z) + u^i(\rho,z) & , \quad \rho < a, z \in (-\infty,\infty) \\ u_2^{(1)}(\rho,z)\,[H(z) - H(z-l)] + u_2^{(1)}(\rho,z)H(z-l), & a < \rho < b, z \in (0,\infty) \end{cases} \tag{1}$$

where $H(z)$ is the unit step function and,

$$u^i(\rho,z) = e^{ikz} \tag{2}$$

is the incident wave with $k = \omega/c$ being the wave number. For the sake of analytical convenience we will assume that the surrounding medium is slightly lossy and k has a small positive imaginary part. The lossless case can be obtained by letting $\mathrm{Im}(k) \to 0$ at the end of the analysis.

The unknown scalar potentials $u_1(\rho,z)$ and $u_2^{(1,2)}(\rho,z)$ satisfy the Helmholtz equation in the regions $z \in (-\infty,\infty)$ and $z \in (0,\infty)$, respectively.

$$\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\right)+\frac{\partial^2}{\partial z^2}+k^2\right]u_{1,2}(\rho,z)=0 \tag{3}$$

together with the boundary conditions and continuity equations:

$$\frac{\partial}{\partial\rho}u_1(a,z)=0\ ,\qquad z<l \tag{4}$$

$$\frac{\partial}{\partial z}u_2^{(1)}(\rho,0)=0\ ,\qquad a<\rho<b \tag{5}$$

$$\left[ik\eta_1+\frac{\partial}{\partial\rho}\right]u_2^{(1)}(a,z)=0\ ,\qquad 0<z<l \tag{6}$$

$$\left[ik\eta_2-\frac{\partial}{\partial\rho}\right]u_2^{(1)}(b,z)=0\ ,\qquad 0<z<l \tag{7}$$

$$\frac{\partial}{\partial\rho}u_2^{(2)}(b,z)=0\ ,\qquad z>l \tag{8}$$

$$\frac{\partial}{\partial z}u_2^{(1)}(\rho,l)-\frac{\partial}{\partial z}u_2^{(2)}(\rho,l)=0\ ,\qquad a<\rho<b \tag{9}$$

$$u_2^{(1)}(\rho,l)-u_2^{(2)}(\rho,l)=0\ ,\qquad a<\rho<b \tag{10}$$

$$\frac{\partial}{\partial\rho}\left[u_1(a,z)+u^i(a,z)\right]=\frac{\partial}{\partial\rho}u_2^{(2)}(a,z)\ ,\ \ z>l \tag{11}$$

$$u_1(a,z)+u^i(a,z)=u_2^{(2)}(a,z)\ ,\quad z>l \tag{12}$$

The above mixed boundary value problem will be solved mainly by using Fourier transform technique together with series expansion of unknown field in overlap region.

2.1. Fourier transformation/Wiener-Hopf equations

Consider the Fourier transform of the Helmholtz equation in the region $\rho < a$ for $z \in (-\infty,\infty)$, namely,

$$\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\right)+(k^2-\alpha^2)\right]F(\rho,\alpha)=0 \tag{13}$$

where $F(\rho,\alpha)$ is the Fourier transform of the field $u_1(\rho,z)$ defined to be

$$F(\rho,\alpha)=\int_{-\infty}^{\infty}u_1(\rho,z)e^{i\alpha z}dz=e^{i\alpha l}[F_+(\rho,\alpha)+F_-(\rho,\alpha)] \tag{14}$$

$F_\pm(\rho,\alpha)$ are half-plane analytical functions on complex α-plane defined by Fourier integrals as:

$$F_\pm(\rho,\alpha)=\pm\int_{l}^{\pm\infty}u_1^e(r,z)e^{i\alpha(z-l)}dz \tag{15}$$

Owing to the analytical properties of $F_{\pm}(\rho,\alpha)$ the solution of (13) reads

$$F_{+}(\rho,\alpha)+F_{-}(\rho,\alpha)=-\dot{F}_{+}(a,\alpha)\frac{J_0(K\rho)}{K(\alpha)J_1(Ka)} \tag{16}$$

where $K(\alpha)=\sqrt{k^2-\alpha^2}$ is the square root function defined by $K(0)=k$. The dot $(\cdot)$ over F represents the derivative with respect to ρ and J_n stands for the Bessel function of integer order.
In the region $a<\rho<b$ for $z\in(l,\infty)$, half Fourier transform of the Helmholtz equation becomes

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+K^2(\alpha)\right]G_{+}(\rho,\alpha)=\frac{\partial}{\partial z}u_2^{(2)}(\rho,l)-i\alpha u_2^{(2)}(\rho,l) \tag{17}$$

where $G_{+}(\rho,\alpha)$ is an analytic function in the upper half of the α-plane which is defined as

$$G_{+}(\rho,\alpha)=\int_l^{\infty}u_2^{(2)}(\rho,z)e^{i\alpha(z-l)}dz \tag{18}$$

The general solution of (17) includes particular part in addition to homogeneous solution. To find the particular solution we use Green's function technique. Without going into detail we obtain the solution of (17) as

$$\begin{aligned}G_{+}(\rho,\alpha)=&-\dot{F}_{+}(a,\alpha)\frac{[J_0(K\rho)KY_1(Kb)-Y_0(K\rho)KJ_1(Kb)]}{K^2(\alpha)[J_1(Ka)Y_1(Kb)-J_1(Kb)Y_1(Ka)]}\\&+\int_l^{\infty}\left[\frac{\partial}{\partial z}u_2^{(2)}(t,l)-i\alpha u_2^{(2)}(t,l)\right]\mathcal{H}(t,\rho,\alpha)\,t\,dt\end{aligned} \tag{19}$$

where $\mathcal{H}(t,\rho,\alpha)$ is a suitable Green's function to determine the particular solution.
Now using continuity relations (11) and (12) we obtain the following equation valid in the strip $Im(-k)<Im(\alpha)<Im(k)$,

$$\dot{F}_{+}(a,\alpha)\frac{N(\alpha)}{K^2(\alpha)}+\frac{a}{2}F_{-}(a,\alpha)=\frac{b}{\pi a}\frac{(f_0-i\alpha g_0)}{k^2-\alpha^2}+\frac{1}{\pi}\sum_{m=1}^{\infty}\frac{J_1(K_m b)}{J_1(K_m a)}\frac{(f_m-i\alpha g_m)}{\alpha_m^2-\alpha^2}-\frac{a}{2}\frac{e^{ikl}}{i(k+\alpha)} \tag{20}$$

which is the Wiener-Hopf equation to be solved through classical procedures. Here, $N(\alpha)$ stands for the kernel function

$$N(\alpha)=\frac{J_1(Kb)}{\pi\big[J_1(Ka)Y_1(Kb)-J_1(Kb)Y_1(Ka)\big]J_1(Ka)}=N_{+}(\alpha)N_{-}(\alpha) \tag{21}$$

Performing standard factorization and decomposition procedures and then applying Liouville's theorem we get the solution of the Wiener-Hopf equation in (20) of the form:

$$\begin{aligned}\dot{F}_{+}(a,\alpha)\frac{N_{+}(\alpha)}{(k+\alpha)}=&-\frac{a}{\pi b}\frac{(f_0+ikg_0)}{(k+\alpha)N_{+}(k)}\\&+\frac{1}{\pi}\sum_{m=1}^{\infty}\frac{J_1(K_m b)}{J_1(K_m a)}\frac{(k+\alpha_m)}{N_{+}(\alpha_m)}\frac{(f_m+i\alpha_m g_m)}{2\alpha_m(\alpha_m+\alpha)}-\frac{ka\,e^{ikl}}{i(k+\alpha)N_{+}(k)}\end{aligned} \tag{22}$$

where $K(\alpha_m)=K_m$ $(m=1,2\ldots)$ are the zeros of the function $K(\alpha)\big[J_1(Ka)Y_1(Kb)-J_1(Kb)Y_1(Ka)\big]$ and f_m, g_m's are expansion coefficients and will be determined later. $N_{+}(\alpha)$ is the split function resulting from the Wiener-Hopf factorization of $N(\alpha)$ and its explicit expression can be found in [10].

2.2. Series Expansion and Determination of Unknown Coefficients

The unknown field $u_2^{(1)}(\rho, z)$ in the region $a < \rho < b$, $0 < z < l$ can be expressed is terms of the waveguide modes as

$$u_2^{(1)}(\rho, z) = \sum_{n=0}^{\infty} a_n \left[e^{i\beta_n z} + e^{-i\beta_n z}\right]\left[J_0(\gamma_n \rho) - R_n Y_0(\gamma_n \rho)\right] \tag{23}$$

with

$$R_n = \frac{ik\eta_1 J_0(\gamma_n a) - \gamma_n J_1(\gamma_n a)}{ik\eta_1 Y_0(\gamma_n a) - \gamma_n Y_1(\gamma_n a)} = \frac{ik\eta_2 J_0(\gamma_n b) + \gamma_n J_1(\gamma_n b)}{ik\eta_2 Y_0(\gamma_n b) + \gamma_n Y_1(\gamma_n b)} \tag{24}$$

where $\eta_{1,2}$ are admittance values related with impedances as $Z_i = \frac{1}{\eta_i}$ $(i = 1,2)$ and γ_n's are the roots of the equation

$$\frac{ik\eta_1 J_0(\gamma_n a) - \gamma_n J_1(\gamma_n a)}{ik\eta_1 Y_0(\gamma_n a) - \gamma_n Y_1(\gamma_n a)} - \frac{ik\eta_2 J_0(\gamma_n b) + \gamma_n J_1(\gamma_n b)}{ik\eta_2 Y_0(\gamma_n b) + \gamma_n Y_1(\gamma_n b)} = 0 \tag{25}$$

while β_n's are defined as

$$\beta_n = \sqrt{k^2 - \gamma_n^2}\ , \quad n = 1,2,\dots \tag{26}$$

Taking into account continuity relations (9) and (10) together with the expression (23) and W-H solution (22), we obtain a set of linear algebraic equations in terms of the unknown coefficients a_n.

$$\begin{aligned}\frac{\pi^2}{2} N_+(k) \sum_{n=0}^{\infty} a_n\left[(k-\beta_n)e^{i\beta_n l} + (k+\beta_n)e^{-i\beta_n l}\right]\Delta_{0n} = \\ -\frac{1}{N_+(k)S_0}\sum_{n=0}^{\infty}\left[a_n(k+\beta_n)e^{i\beta_n l} + (k-\beta_n)e^{-i\beta_n l}\right]\Delta_{0n} \\ -k\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[a_n(\alpha_m+\beta_n)e^{i\beta_n l} + (\alpha_m-\beta_n)e^{-i\beta_n l}\right]\frac{\Delta_{mn}}{\alpha_m N_+(\alpha_m) S_m} - \frac{\pi k a e^{ikl}}{N_+(k)}\end{aligned} \tag{27}$$

$$\begin{aligned}\frac{\pi^2}{2} N_+(\alpha_r) \sum_{n=0}^{\infty}\left[a_n(\alpha_r-\beta_n)e^{i\beta_n l} + (\alpha_r+\beta_n)e^{-i\beta_n l}\right]\Delta_{rn} = \\ -\frac{1}{N_+(k)S_0}\sum_{n=0}^{\infty}\left[a_n(k+\beta_n)e^{i\beta_n l} + (k-\beta_n)e^{-i\beta_n l}\right]\Delta_{0n} \qquad (r = 1,2,\dots) \\ -\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[a_n(\alpha_m+\beta_n)e^{i\beta_n l} + (\alpha_m-\beta_n)e^{-i\beta_n l}\right]\frac{(k+\alpha_m)\Delta_{mn}}{2\alpha_m N_+(\alpha_m) S_m}\frac{(k+\alpha_r)}{(\alpha_m+\alpha_r)} - \frac{\pi k a e^{ikl}}{N_+(k)}\end{aligned} \tag{28}$$

where

$$S_0 = \frac{2}{\pi^2}\frac{a^2 - b^2}{a^2} \quad , \quad S_m = \frac{2}{\pi^2}\frac{J_1^2(K_m a) - J_1^2(K_m b)}{J_1^2(K_m b)} \tag{29}$$

$$\Delta_{0n}=\frac{2}{\pi\gamma_n}\left\{[J_1(\gamma_n a)-R_nY_1(\gamma_n a)]-\frac{b}{a}[J_1(\gamma_n b)-R_nY_1(\gamma_n b)]\right\} \tag{30}$$

$$\Delta_{mn}=\frac{2}{\pi}\frac{\gamma_n}{\gamma_n^2-K_m^2}\left\{[J_1(\gamma_n a)-R_nY_1(\gamma_n a)]-\frac{J_1(K_m a)}{J_1(K_m b)}[J_1(\gamma_n b)-R_nY_1(\gamma_n b)]\right\} \tag{31}$$

Solving the above set of equations (27-28) numerically we determine f_m and g_m from the relations.

$$f_0+i\alpha g_0=\frac{i}{S_0}\sum_{n=0}^{\infty}\left[a_n(\alpha+\beta_n)e^{i\beta_n l}+(\alpha-\beta_n)e^{-i\beta_n l}\right]\Delta_{0n} \tag{32}$$

$$f_m+i\alpha g_m=\frac{i}{S_m}\sum_{n=0}^{\infty}\left[a_n(\alpha+\beta_n)e^{i\beta_n l}+(\alpha-\beta_n)e^{-i\beta_n l}\right]\Delta_{mn} \tag{33}$$

2.3. Reflection and Transmission Coefficients

The scattered field $u_1(\rho,z)$ can be obtained by taking the inverse Fourier transform of $F(\rho,\alpha)$. From the definition (14) and solution (16) we can write,

$$u_1(\rho,z)=-\frac{1}{2\pi}\int_{-\infty}^{\infty}\dot{F}_+(a,\alpha)\frac{J_0(K\rho)}{K(\alpha)J_1(Ka)}e^{-i\alpha(z-l)}d\alpha \tag{34}$$

The evaluation of this integral for $z < l$ and $z > l$ will give us the reflected wave and the transmitted wave, respectively. The reflection coefficient R of the fundamental mode is defined as the complex coefficient of the term exp(-ikz) and is computed from the contribution of the first pole at $\alpha=k$. The result is

$$R=-\frac{e^{i2kl}}{[N_+(k)]^2}-\frac{i}{\pi}\frac{(f_0+ikg_0)}{kb[N_+(k)]^2}e^{ikl}+\frac{i}{\pi}\frac{e^{ikl}}{aN_+(k)}\sum_{m=1}^{\infty}\frac{J_1(K_m b)}{J_1(K_m a)}\frac{(f_m+i\alpha_m g_m)}{\alpha_m N_+(\alpha_m)} \tag{35}$$

Similarly, the transmission coefficient T of the fundamental mode which is defined as to be the complex coefficient of exp(ikz) is calculated from the contribution at the pole $\alpha=-k$ as,

$$T=\frac{a^2}{b^2}+\frac{ie^{-ikl}}{\pi ka}\left(\frac{b}{a}-\frac{a}{b}\right)(f_0+ikg_0) \tag{36}$$

3. Results and Discussion

In order to show the effects of the parameters like the length of the extended inlet l and the surface admittances $\eta_{1,2}$ on the sound transmission, some numerical results showing the variation of transmission coefficient T are presented. In all numerical calculations the solution of the infinite system of algebraic equations is obtained by truncating the infinite series at N = 5 , since the transmission coefficient becomes insensitive for N > 5. We also limit ourselves with only imaginary values of surface admittances for simplicity. Such that $\eta_{1,2}=iX_{1,2}$, $X\in\mathbb{R}$.

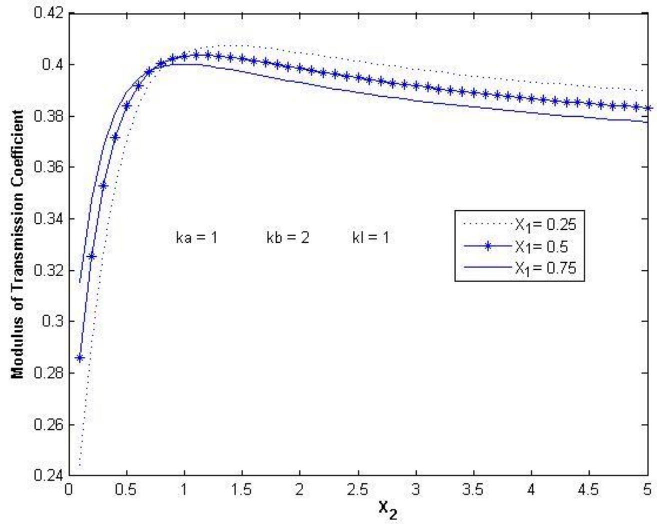

Figure 2. Transmission coefficient *T* versus the surface admittance $\eta_2 = iX_2$, $X_2 > 0$ for different values of $\eta_1 = iX_1$

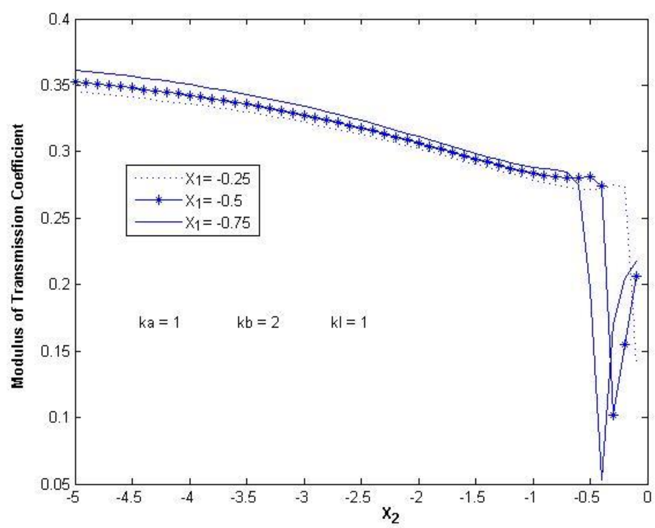

Figure 3. Transmission coefficient *T* versus the surface admittance $\eta_2 = iX_2$, $X_2 < 0$ for different values of $\eta_1 = iX_1$

In Fig. 2 and Fig. 3, as the admittance X_2 of the lateral wall of the expanding duct increases the transmitted field is ascending until at some value of X_2 then it starts to attenuate gradually. But for negative values of X_2 the attenuation is more visible especially around $-0.5 < X_2 < 0$. For different values of X_1 some decrease in the transmitted field is observed.

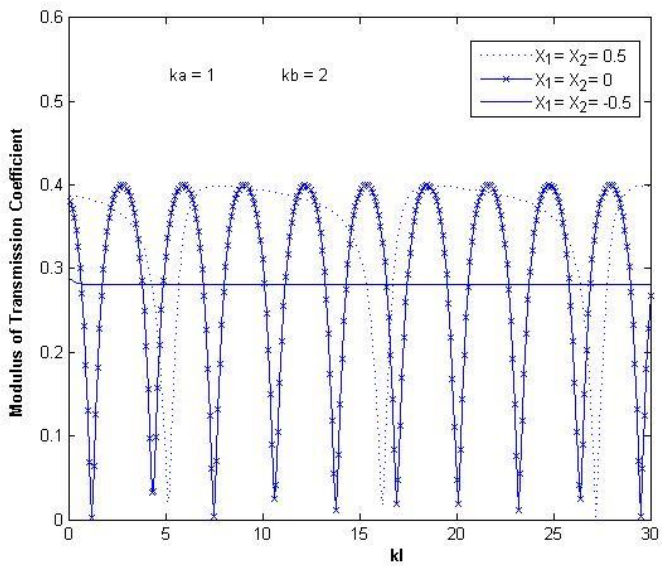

Figure 4. Transmission coefficient *T* versus the extended inlet length *kl* for different values of X_1 and X_2.

In Fig. 4, an oscillatory behaviour is seen for increasing values of the extended inlet length kl, but this behaviour is broken for negative values of X_1 and X_2.

Conclusions

This paper examines the transmission of sound waves in an extended tube resonator whose walls are treated by acoustically absorbing materials in overlap region. To analyse the problem a hybrid method of formulation consisting of expressing the total field in terms of complete sets of orthogonal waveguide modes and using the Fourier transform elsewhere is adopted. The mixed boundary value problem is reduced to a Wiener-Hopf equation whose solution involves infinitely many expansion coefficients satisfying a system of linear algebraic equations. These equations are solved numerically and the effects of various problem parameters on transmitted field are displayed graphically. It was seen that adjusting the impedance values in overlap region it is possible to control the amount of the transmission from inner duct into the outer one within reasonable bounds. As a future work a similar problem now with an extended inlet will be studied following the same method used in here.

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